MATH 211 Basic Algebra

Problem Set 2

1. Multiply permutations in the given order and in the inverse order:

   (a)
$$\begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 3 & 4 & 1 & 5 & 2\end{pmatrix}, \qquad \begin{pmatrix}1 & 2 & 3 & 4 & 5\\ 5 & 3 & 1 & 2 & 4\end{pmatrix}$$

   (b)
$$\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 3 & 6 & 4 & 5 & 2 & 1\end{pmatrix}, \qquad \begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 2 & 4 & 1 & 5 & 6 & 3\end{pmatrix}$$

2. Write each of the four permutations above as a product of disjoint cycles.
3. Write each of the four permutations above as a product of transpositions.
4. Write each of the following permutations in the form of table: $(136)(247)(5)$, (1654237), $(135\ldots 2n-1)(246\ldots 2n)$.
5. Multiply the first two permutations from the previous problem in the given and in the inverse order.
6. Find the number of inversions for the each of the permutations in the first problem and determine their parity.
7. Find the sign of the following permutations:
   (a) $(123\ldots k)$
   (b) $(i_1 i_2 i_3 \ldots i_k)(j_1 j_2 \ldots j_m)$
   (c) Show that every permutation in S_n can be represented a product of transpositions of the form $(12), (13), \ldots, (1n)$.
   (d) Show that every permutation in S_n can be represented a product of several permutations each of which is equal to (12) or $(123\ldots n)$.
8. Show that a permutation in S_n is even iff it can be represented as a product of several cycles of length 3.