MATH 211
BASIC ALGEBRA 1
Final exam
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1. Show that the polynomial $x^7 + \frac{3}{2}x^5 - 3x + \frac{3}{2}$ is irreducible over $\mathbb{Q}$.
2. Find all rational roots of the polynomial $4x^4 - 7x^2 - 5x - 1$.
3. Find the greatest common divisor of the polynomials over the two-element field $\mathbb{F}_2$
$$x^5 + x^4 + 1 \quad \text{and} \quad x^4 + x^2 + 1$$
and its linear expression in terms of them.
4. Prove that the quotient-rings $\mathbb{Q}[x]/(x^2 - 2)\mathbb{Q}[x]$ and $\mathbb{Q}[x]/(x^2 - 3)\mathbb{Q}[x]$ are fields. Are they isomorphic?
5. Prove that a finite associative commutative ring without zero divisors is a field.
6. Find all subgroups of the group S_3. Which of them are normal?
7. Find all homomorphisms from
   (a) the group $\mathbb{Z}_{10}$ to the group S_3,
   (b) the group S_3 to the group $\mathbb{Z}_{10}$.