

Math 221: Problem Set #3

Solutions to the problems must be presented in the written form to the instructor at 11/01/2001.

Find the limits:

1.

$$\lim_{x \rightarrow 0} \frac{\sqrt[m]{1 + \alpha x} \sqrt[n]{1 + \beta x} - 1}{x},$$

where m and n are natural numbers.

2.

$$\lim_{x \rightarrow 1} \frac{(1 - \sqrt{x})(1 - \sqrt[3]{x}) \dots (1 - \sqrt[n]{x})}{1 - x^{n-1}}.$$

3.

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1 + x^2} + x)^n - (\sqrt{1 + x^2} - x)^n}{x},$$

where $n \in \mathbf{N}$.

4.

$$\lim_{z \rightarrow \pi} \frac{\sin mx}{\sin nx},$$

where $m, n \in \mathbf{N}$.

5.

$$\lim_{z \rightarrow 0} \frac{1 - \cos x}{x^2}.$$

6.

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}.$$

7.

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a}.$$

8.

$$\lim_{x \rightarrow 0} \frac{\cos x - \cos 3x}{x^2}.$$

9. Find real numbers α, β from the following condition:

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x + 1} - \alpha x - \beta \right) = 0.$$

Find the limits:

10.

$$\lim_{x \rightarrow 0} \left(\frac{\cos x}{\cos 2x} \right)^{1/x^2}.$$

11.

$$\lim_{x \rightarrow 0} \frac{\ln \cos ax}{\ln \cos bx}.$$

12.

$$\lim_{x \rightarrow \infty} \arcsin \frac{1 - x}{1 + x}.$$