MATH 111

Homework 8

Solutions

1. Let X and A be two sets.

   (a) Define the "object" $B := \{y \cap A \mid y \in X\}$. This object is a set because we have

   $$B = \{y \in P(\cup X) \mid \exists z\ z \in X\ \wedge\ (\forall t\ t \in y \longleftrightarrow t \in a \wedge t \in z)\}$$

   (b) Come on, this is easy! (Hint: Draw it!)

2. Let X be a set. Let us show that the collection S of two element subsets of X form a set:

   $$S = \{y \in P(X) \mid \exists u\ \exists v\ u \in y\ \wedge v \in y \wedge u \neq v \wedge (\forall t\ t \in y \rightarrow t = u \vee t = v)\}$$

3. Done in class.

4. Done in class.

5. Done in class.

6. Done in class.

7. Done in class.

8. Done in class.

9. Let us show that the collection S of subsets of $\mathbb{N}$ which contain an odd number form a set:

   $$S = \{A \in P(\mathbb{N}) \mid \exists k\ \ \exists n\quad k \in A \wedge\ \ n \in N\ \ \wedge k = 2n + 1\ \}$$

10. Let us show that every nonempty subset of $\mathbb{N}$ has a smallest element. Let A be a subset of $\mathbb{N}$ which does not contain a smallest element. We will show that A is empty. Look at the complement A^c of A. Since A has no smallest element it cannot contain 0. Thus $0 \in A^c$. Let n be a nonzero number. Assume $n \notin A$ (hence all numbers smaller than n are also not in A) then $n + 1 \notin A$ because otherwise $n + 1$ would be the smallest element of A. The last two sentences mean that if $n \in A^c$ then $n + 1 \in A^c$. Hence A^c is an inductive set. Since $\mathbb{N}$ is the smallest inductive set $A^c = \mathbb{N}$, hence $A = \emptyset$ as desired.