

MATH 111

Homework 11

Notations: AC denotes the Axiom of Choice, ZL denotes Zorn's Lemma.

1. Find an element of $\prod_{\substack{X \subseteq \mathbb{Q} \\ X \neq \emptyset}} X$.
2. Fill the following gaps in the proof of the fact that AC implies ZL :
 - (a) For any chain C in X , show that $\{A \in T_0 : A \subseteq C \text{ or } g(C) \subseteq A\}$ is a tower.
 - (b) Deduce from a) that the set of comparable elements of T_0 is a tower.
 - (c) Deduce from b) that T_0 is totally ordered.
 - (d) Deduce from c) that $\bigcup T_0$ is a maximal chain in X .