

MATH 111

Homework 3

1. For any set X , let $P(X)$ denote the set of all subsets of X . Which of the following equations are true? If the equation is not true, is one of the sides a subset of the other? Prove your answers:

(a) $\bigcup_{A \in P(X)} A = X$

(b) $P(X \cap Y) = P(X) \cap P(Y)$

(c) $P(X \cup Y) = P(X) \cup P(Y)$

(d) $P(X \times Y) = P(X) \times P(Y)$ where X and Y are arbitrary sets.

2. Let R be a relation on a set X . Assume that R is reflexive and transitive.

- (a) Define a relation S on X by

$$S = \{(x, y) \mid (x, y) \in R \text{ and } (y, x) \in R\}$$

Is S an equivalence relation on X ? Prove your answer.

- (b) Define a relation T on X by

$$T = \{(x, y) \mid (x, y) \in R \text{ or } (y, x) \in R\}$$

Is T an equivalence relation on X ? Prove your answer.

3. Let X be a set and let R be the relation defined on $P(X)$ as follows

$$(A, B) \in R \iff A \cap B \neq \emptyset$$

Is R :

- (a) reflexive?
(b) symmetric?
(c) transitive?

Prove your answers.