

MATH 111

Homework 4

1. Let f be a function between two sets:

$$f : X \longrightarrow Y$$

Define a relation \sim on X by

$$x \sim y \iff f(x) = f(y)$$

- (a) Show that " \sim " is an equivalence relation on X .
(b) Let $Z = X/\sim$ be the set of equivalence classes in X defined by the relation \sim , hence

$$Z = \{[x] \mid [x] = \text{equivalence class of } x, \text{ for all } x \in X\}$$

Show that f defines a unique function $F : Z \longrightarrow Y$ by the formula

$$F([x]) = f(x), \quad \text{for all } [x] \in Z.$$

- (c) Show that F is one-to-one and the function f and F have the same range set.
2. Let $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be two functions. Let $h = g \circ f : X \longrightarrow Z$. Show that
- (a) If f and g are one-to-one, so is h .
(b) If f and g are onto, so is h .
(c) If f and g are bijective, so is h .
(d) Let f and g be bijective. Show that the inverse functions f^{-1}, g^{-1}, h^{-1}

verify:

$$h^{-1} = f^{-1} \circ g^{-1}$$

3. Let $F : A \longrightarrow B$ and $G : B \longrightarrow C$ be two functions and let $H : G \circ F : A \longrightarrow C$. Show that
- (a) If H is one-to-one, so is F .
(b) If H is onto, so is G .
(c) If H is bijective, F and G are not necessarily bijective. Give a counter example also.
4. Show that a transitive, symmetric relation R on a set X is not necessarily reflective.