MATH 111
Homework 6

1. Prove Bernoulli's Inequality; let $x > -1$, show that *for each natural number* $n \geq 1$
$$(1+x)^n \geq 1 + n.x$$

2. Let $a_n = 2 - \frac{1}{2^n}$. Show that $\lim_{n\to\infty} a_n = 2$. Hint: Use Bernoulli's Inequality.

3. Prove that $\lim_{n\to\infty} \frac{2n^2+1}{n^2+3n} = 2$.

4. Give examples of :

   (a) A convergent sequence $(a_n)_{n\in\mathbb{N}}$ and a divergent sequence $(b_n)_{n\in\mathbb{N}}$ such that $(a_n.b_n)_{n\in\mathbb{N}}$ is convergent,

   (b) A pair of divergent sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ such that $(a_n + b_n)_{n\in\mathbb{N}}$ is convergent,

   (c) A pair of divergent sequences $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ with $(a_n.b_n)_{n\in\mathbb{N}}$ convergent.

5. Prove that if $\lim_{n\to\infty} a_n = 0$ and $(b_n)_{n\in\mathbb{N}}$ is bounded than $\lim_{n\to\infty} a_n.b_n = 0$.

6. Prove that if $(a_n)_{n\in\mathbb{N}}$ converges and $(b_n)_{n\in\mathbb{N}}$ diverges then $(a_n + b_n)_{n\in\mathbb{N}}$ diverges.