MATH 111
Homework 7

1. Show that $\mathbb{Z} \times \mathbb{Z}$ with the lexicographic order is not order isomorphic to $\mathbb{Z}$ with its usual order.

2. Show that a sequence is Cauchy iff $\lim\limits_{n,m\to\infty} |x_n - x_m| = 0$.

3. Find examples of sequences $(a_n)_n$ and $(b_n)_n$ such that $\lim \frac{1}{a_n} = 0$ and $\lim b_n = 0$ and

   (a) $\lim a_n.b_n = 0$,

   (b) $\lim (a_n.b_n)^{-1} = 0$,

   (c) $\lim (a_n.b_n)$ does not exist and, $\lim (a_n.b_n)^{-1}$ does not exist,

   (d) $\lim a_n.b_n = c$ where c is an arbitrary given real number.

4. Find the limit of each of the following sequences.

   (a) $a_1 = 1$ and $a_n = \sqrt{1 + a_{n-1}}$, $n = 2, 3, ...$

   (b) $a_1 = \sqrt{2}$ and $a_n = \sqrt{2a_{n-1}}$, $n = 2, 3, ...$

   (c) $\left(1 + \frac{1}{n^2}\right)^2$

   (d) $\left(1 + \frac{1}{2n}\right)^n$

5. Let E be a set. Define the binary relation $I_E = \{(e, e) \mid e \in E\}$ and for any binary relation R define $R^{-1} = \{(x, y) \mid (y, x) \in R\}$. Show that if R is transitive then $I_E \cup \left(R \cap R^{-1}\right)$ is an equivalence relation on E.