

MATH 111
2001-2002
Homework 8
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1. Let x and a be two sets. Recall that $\cap x$ means the intersection of all sets which are in x , i.e. $\cap x = \{z : z \in y \text{ for all } y \in x\}$.
 - (a) Prove that "the object" $b := \{y \cap a : y \in x\}$ is a set.
 - (b) Show that $a \cap (\cap x) = \cap b$.
2. Let x be a set. Show that the elements of x with exactly two elements form a set.
3. Cartesian Product of Two Sets. Let x and y be two sets. Show that "the object" $\{\{a\}, \{a, b\}\} : a \in x, b \in y$ is a set.
4. Show that if x is a set, then $x \cup \{x\}$ is also a set. The set $x \cup \{x\}$ is sometimes denoted $x + 1$.
5. A set x is called *inductive* if $\emptyset \in x$ and if for all $y \in x$, the set $y + 1$ (defined above) is also an element of x . Write a formula $\phi(v)$ in the language of set theory such that $\phi(v)$ holds for a set x if and only if x is an inductive set. (in other words, write down what it means to be inductive with our symbols_Haluk)
6. Let A be a set whose elements are inductive sets. Show that $\cap A$ is also an inductive set.
7. From the axioms of set theory that we have seen up to now, we cannot prove that there is an inductive set. Assuming there is an inductive set, prove that there is an inductive set (denoted by \mathbb{N}) which is a subset of every inductive set.
8. Do you have any idea what \mathbb{N} can be?
9. Show that the subsets of \mathbb{N} that contain an odd number form a set.(you may write $a \mid b$, when you want to write "a divides b".)
10. Show that every nonempty subset of \mathbb{N} has a least element.