

MATH 111

Quiz 4

January 2002

1. Let $X = \{f : \mathbb{R} \rightarrow \mathbb{R}\}$. For $f, g \in X$, define

$$f \sim g \quad \text{iff} \quad \exists \varepsilon > 0 \text{ s.t. } \forall x \in (-\varepsilon, +\varepsilon) \quad f(x) = g(x)$$

- (a) Show that \sim is an equivalence relation.
 - (b) Is it true that for any $f \in X$, $[f] = [|f|]$? Justify your answer.
 - (c) Let $Z = \{[f] \mid f \in X\}$. Let $F : Z \rightarrow \mathbb{R}$ be defined by $F([f]) = f(0)$. Show that F is well-defined and F is not 1-1.
2. Let $(x_n)_{n \in \mathbb{N}}, (y_n)_{n \in \mathbb{N}}$ be two sequences. Assume that there is $N \in \mathbb{N}$ s.t. for all $n > N$ $x_n = y_n$.
- (a) Show that if $(x_n)_{n \in \mathbb{N}}$ is bounded, then $(y_n)_{n \in \mathbb{N}}$ is bounded.
 - (b) Show that if $(x_n)_{n \in \mathbb{N}}$ is Cauchy, then $(y_n)_{n \in \mathbb{N}}$ is Cauchy.
 - (c) If $(x_n)_{n \in \mathbb{N}}$ converges to some number A , then $(x_n)_{n \in \mathbb{N}}$ converges to A too.